The leading term in the second-order lift coefficient expansion is

$$C_L = -\frac{3}{16}\epsilon^2 h^{-3} \tag{26}$$

The lift coefficient for the first-order angle-of-attack problem is

$$C_L = 2\pi\alpha\left(1+\frac{1}{4}h^{-2}-\frac{3}{32}h^{-4}-\frac{1}{512}h^{-6}+0(h^{-8})\right) \tag{27}$$

Note that h^{-1} is the semichord-to-wall clearance ratio.

It is important to note that this problem contains two small parameters, one associated with the airfoil disturbance (ϵ,α) and one associated with the wall disturbance (h^{-1}). Consider the first-order lift coefficient due to angle of attack [Eq. (27)]. The series converges rapidly in h^{-1}. The expansion in h^{-1} of the exact solution of Havelock[2] is

$$C_L = 2\pi\sin\alpha\Big\{1-\sin\alpha h^{-1}+\frac{1}{4}h^{-2}[1+3\sin^2\alpha]$$
$$-\frac{1}{4}h^{-3}[\sin\alpha+3\sin^3\alpha]$$
$$-\frac{3}{32}h^{-4}[3-13\sin^2\alpha-22\sin^4\alpha]+0(h^{-5})\Big\} \tag{28}$$

The terms linear in α in Eqs. (27) and (28) are identical.

Consider the first-order lift coefficient due to thickness [Eq. (25)]. For a distance from the wall of greater than one chord $(h=2)$ better than 1% accuracy is achieved with only one term. Green[3] presents a series expansion for the Joukowski airfoil at angle of attack but includes no terms higher than second order in h^{-1} so that a comparison cannot be made.

Equations (25) and (27) present results linear in ϵ and α and higher order in h^{-1}. The utility of these results depends upon the magnitude of the nonlinear terms that have been neglected. The leading term in the second-order thickness problem is given in Eq. (26) and is of $0(\epsilon^2 h^{-3})$. The leading nonlinear angle-of-attack terms are obtained from Eq. (28) and are of $0(\alpha^3)$ and $0(\alpha^2 h^{-1})$. The leading interaction terms are given in the expansions of Green[3] and are of $0(\epsilon\alpha)$ and $0(\epsilon\alpha h^{-2})$.

For the linear results presented in this paper, the lift force experienced by the airfoil in an infinite fluid $(h\to\infty)$ is decreased by the effect of thickness and increased by the effect of angle of attack.

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Control Laws for Adaptive Wind Tunnels

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Introduction

THE concept of modifying the conditions at wind tunnel walls on a systematic basis to minimize the interference about a model has been pursued by Sears et al.[1-4] and Lo and Kraft.[5] In this Note, a control law is developed for achieving this end. The approach is general and does not depend on the mechanical means used to measure the flow variables (sensors) or to control the flow variables (actuators). Also, unlike previous control schemes, the present one does not require an iterative approach per se. However, it should be noted that Lo and Kraft have developed a so-called "one-step iteration" method for two-dimensional, nonlifting flows using classical subsonic theory. By contrast the present approach is valid for both three-dimensional and lifting flows and, subject to the assumption of linearity, is not dependent on any particular theoretical description of the fluid.

Technical Discussion

Consider a model in a wind tunnel (Fig. 1). Conceptually enclose the model in a control box, two of whose walls coincide with those of the wind tunnel. It is desired to control the flow variables on the walls of the box so that, as far as the flow in the neighborhood of the model is concerned, it is as though the model were in a fluid of infinite extent. The flow outside the box (which exists only in the form of a mathematical model) will be called the exterior flow; the flow inside the box is called the interior flow.

Exterior Flow

From a mathematical model of the exterior flow (the reader may wish to think of this as a classical potential flow model for definiteness, although the discussion is not limited to such a mathematical model), the relationship between normal velocity V_D and pressure P_D on the box wall may be determined. The subscript D is used to denote these as the values on the box wall which obey the desired relationship to simulate a fluid of infinite extent. For V_D and P_D measured at a certain number of discrete points, this relationship may be written in matrix form as

$$\{V_D\} = [E]\{P_D\} \tag{1a}$$

E is an influence coefficient matrix which gives the value of V_D at one point due to a unit P_D at another point. In practice it may be easier to determine the inverse of E, i.e., P_D at one

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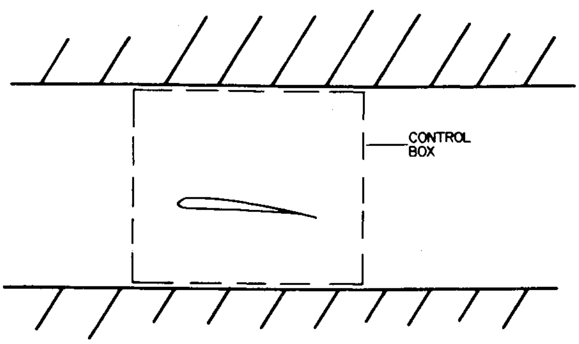

Fig. 1 Model inside wind tunnel and control box.

point due to V_D at another and then invert to obtain E. For potential flow, indeed inviscid flow, Eq. (1a) is uniquely known for a given mathematical model. For a viscous flow model, it is not, of course.

Two issues deserve emphasis. First, the choice of normal velocity and pressure in Eq. (1a) is not essential. Any two variables which are uniquely connected by a relationship of the form of Eq. (1a) could be used, e.g., the normal and tangential velocity components. Second, the values of V_D and P_D are not *separately* prescribed; all that is required is that their relationship to each other be of the form prescribed by Eq. (1a).

Interior Flow

Consider now the interior flow with the model at some reference condition where interference effects are negligible. Let $\{\delta\}$ be the values of the control variable at the points where V and P are to be measured on the wall. By systematically giving each control variable a (unit) value, the increment in normal velocity ΔV and pressure ΔP due to a change in δ may be measured. The results may be expressed in matrix form as

$$\{\Delta V\} = [I_{V\delta}]\{\delta\} \tag{2a}$$

$$\{\Delta P\} = [I_{P\delta}]\{\delta\} \tag{3a}$$

where $I_{V\delta}$ and $I_{P\delta}$ are measured influence matrices. Equations (2a) and (3a) assume a linear relationship between ΔV, ΔP, and δ. More will be said of this later. Also it is assumed that the control locations are chosen such that $[I_{V\delta}]$ and $[I_{P\delta}]$ are nonsingular. Standard mathematical tests can be performed on these matrices to insure that this is the case.

Matching of Interior and Exterior Flows

Consider now the internal flow with the model changed to a new condition from its reference condition but with $\delta = 0$. One measures the nominal velocity and pressure on the wall, call these $\{V_N\}$ and $\{P_N\}$.

If one now applies various control variables δ, then the total velocity and pressure using Eqs. (2a) and (3a) are

$$\{V_N\} + \{\Delta V\} = \{V_N\} + [I_{V\delta}]\{\delta\} \tag{4a}$$

$$\{P_N\} + \{\Delta P\} = \{P_N\} + [I_{P\delta}]\{\delta\} \tag{5a}$$

One now seeks to determine the δ such that the total velocity and pressure are equal to the values V_D and P_D, which obey Eq. (1a), i.e., one requires

$$\{V_N\} + \{\Delta V\} = \{V_D\} \tag{6}$$

$$\{P_N\} + \{\Delta P\} = \{P_D\} \tag{7}$$

Substituting Eqs. (4a), (5a), (6), and (7) into Eq. (1a), one may construct

$$\{V_N\} + [I_{V\delta}]\{\delta\} = [E][\{P_N\} + [I_{P\delta}]\{\delta\}] \tag{8}$$

Solving Eq. (8) for δ gives

$$\{\delta\} = [[I_{V\delta}] - [E][I_{P\delta}]]^{-1}[[E]\{P_N\} - \{V_N\}] \tag{9}$$

Equation (9) is the principal result of this Note. Note that no explicit control variable has been assumed. However, as an example, if δ was identified as an *imposed* normal velocity at the box wall, then the elements of the influence matrix $I_{V\delta}$ would be the fluid velocity at one point due to an imposed velocity at another point and the elements of $I_{P\delta}$ would be the fluid pressure of one point due to a unit velocity imposed at another point.

Before closing two important, although subsidiary, issues are explored:

1) The number of measurement points exceeds the number of control points.

2) Departures of the ΔV and ΔP vs δ relationships from linearity.

When the number of measurement points exceeds the number of control points (the expected condition in practice), then the desired conditions at the box wall can be satisfied only in a least-squares (or some similar) sense. The analysis is briefly outlined here. It is convenient to revert to summation rather than matrix notation.

Thus Eqs. (1a), (2a), and (3a) become

$$V_{D_i} = \sum_{j=1}^{M} E_{ij} P_{D_j} \quad i = 1,\ldots,M \tag{1b}$$

$$\Delta V_i = \sum_{k=1}^{C} I_{V\delta_{ik}} \delta_k \quad i = 1,\ldots,M \tag{2b}$$

$$\Delta P_j = \sum_{k=1}^{C} I_{P\delta_{jk}} \delta_k \quad j = 1,\ldots,M \tag{3b}$$

where M is the number of measurement points, C the number of control points, and $M > C$. Now define the total velocity and pressure as

$$V_i \equiv V_{N_i} + \Delta V_i = V_{N_i} + \sum_{k=1}^{C} I_{V\delta_{ik}} \delta_k \tag{4b}$$

$$P_j \equiv P_{N_j} + \Delta P_j = P_{N_j} + \sum_{k=1}^{C} I_{P\delta_{jk}} \delta_k \tag{5b}$$

Also define a mean square performance index as

$$PI \equiv \sum_{i=1}^{M} \left[V_i - \sum_j E_{ij} P_j \right]^2 \tag{10}$$

Note that PI is a function of the control variables δ from Eqs. (4a) and (5a). The control law for δ can now be determined by requiring that the δ be such that PI is a minimum.

The relationship between ΔV, ΔP, and δ has been assumed to be linear. (We have also assumed the relationship between V_D and P_D to be linear; however this assumption is likely to be less restrictive in practice. Hence the possible nonlinearity of the interior flow is the focus here.) Consider Eqs. (2a) and (3a). These are to be obtained from measurement. At some value of δ, either ΔV or ΔP will (first) depart from linearity with respect to δ. Thus one knows over what range of δ the ΔV vs δ and ΔP vs δ relationships are linear. Hence, a posteriori one can examine the solution for δ to see if it is within the range of linear behavior. If it is not, then a new set of influence coefficient, $I_{V\delta}$, $I_{P\delta}$, would be measured and the process repeated, i.e., a new solution for new increments in δ would be obtained. Of course, there is no guarantee that such a scheme will converge in general. However, if a solution for one flow condition has been obtained, then sufficiently small incremental changes in, say, Mach number or angle of attack would insure that the needed δ are small enough for linear behavior to prevail.

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Computation of Airfoil Buffet Boundaries

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Introduction

UNSTEADY flows about the components of an aircraft that is flying at transonic speeds can lead to buffet.[1] These unsteady flows usually are caused by separated flows or by shock-wave and boundary-layer interactions on wings, either (or both) of which can excite large oscillations in the flowfields and consequently create fluctuating lifting or buffeting forces.[2] Because the upper Mach-number and lift-coefficient limits of transonic aircraft performance are often associated with unsteady forces, an important consideration in choosing an airfoil section, for example, whether conventional or supercritical, is not only the relative merits of its static aerodynamic characteristics but also its relative unsteady-flow characteristics. Consequently, transonic, turbulent, unsteady flows about conventional and supercritical airfoils have been studied experimentally and computationally (see, for example, Refs. 3-6).

Recent results from Refs. 5 and 6 show that computer solutions of the time-dependent, two-dimensional Reynolds-averaged form of the compressible Navier-Stokes equations provide remarkably good comparisons with the essential features of transonic, turbulent, unsteady flows about airfoils, even when the computer code employs a simple algebraic eddy-viscosity turbulence model developed for steady flows. This fact, plus the need by aircraft designers for information about the unsteady-flow characteristics of airfoils, encouraged the present study in which the Mach-number, lift-coefficient boundaries above which unsteady flows occur were computed. The computer code did not include any aeroelastic coupling between the airfoil and the flowfield; however, the computed boundaries will be referred to herein as "buffet boundaries."

This Note presents the results of an initial effort to compute the buffet boundaries for a conventional and a supercritical airfoil and to compare the results with experiment. Changes in the turbulence model are suggested, the implementation of which should provide improved agreement between computed and experimental boundaries.

Approach

It was demonstrated in Ref. 7 that the implicit finite-difference algorithm described in Ref. 6 for solving the "thin-layer" Navier-Stokes equations could be programmed for the ILLIAC IV computer and used in a practical manner to compute the aileron buzz phenomenon. The same computer and algorithm, without provisions for aileron motion, were used for the present study, in which many solutions for various combinations of Mach number and angle of attack were required. A two-layer eddy-viscosity turbulence model was used in which the outer-layer length scale is determined as a function of the local vorticity.[8] Turbulent flow was initiated at the airfoil leading edge ($x/c=0$).

There is a paucity of data concerning buffet boundaries of airfoils at high Reynolds numbers. However, such information was reported in Ref. 3 for an NACA 65_1-213, $a=0.5$ conventional airfoil at a chord Reynolds number $Re_c=17\times10^6$ and for a Garabedian-Korn supercritical airfoil at $Re_c=21\times10^6$. To reduce tunnel wall interference effects, the results for these airfoils with natural boundary-layer transition were obtained in a porous-wall, transonic wind tunnel, with a wall porosity of 20.5%. Since porous-wall boundary conditions cannot be simulated numerically without knowledge of the flow through the walls, the computations were made using free-flight boundary conditions. The experimental buffet boundaries for the conventional airfoil were defined by observing the divergence in the upper surface trailing-edge pressure at $x/c=0.95$ and the output signal from a side wall force balance; those for the supercritical airfoil were determined from the force-balance signal alone. The computed buffet boundaries were defined, for a given freestream Mach number M_∞, as the mean lift coefficient C_ℓ at which an unsteady lift (flowfield) was first obtained from solutions at increasing increments in angle of attack α (to the nearest ¼ deg). For the supercritical airfoil at $M_\infty=0.60$ and 0.75, it was necessary to obtain solutions at α's to the nearest ⅛ deg to verify the initial solution for unsteady lift.

Results and Discussion

Experimental and computed buffet boundaries for the conventional airfoil section are shown in Fig. 1. Computed boundaries are for an NACA 65-213, $a=0.5$ airfoil for which the coordinates are negligibly different from the NACA 65_1-213, $a=0.5$ airfoil. Limited experimental data[3] were obtained at the lower lift coefficients and higher Mach numbers. To extend the data to higher lift coefficients and lower Mach numbers, the flight-determined buffet boundary for the F-80A airplane[1] is also shown. The F-80A was a straight-wing airplane with an NACA 65-213, $a=0.5$ airfoil section; the boundary was defined using accelerometer data. The flight Reynolds numbers, not specified in Ref. 1, were estimated from the listed flight speeds and altitudes; they range from 12×10^6 to 17×10^6. The agreement between the two sets of

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